

Laboratory 2

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EXERCISE 1 (25 POINTS)

Symbolic integration. MATLAB supports operations with symbolic variables, that is, all kinds of calculations using symbols in place of real-valued (numerical) variables. Symbolic integration is implemented in MATLAB through function `int()`. Write your own function named `integral()` that invokes `int` and has the following input data `[integral(f,t,r,a,b)]` in order to compute the integral $\int_a^b f t dr$: f and t (their product, represents the function to be integrated), r (independent variable of integration), and a and b (integration limits).

EXERCISE 2 (25 POINTS)

In this exercise we determine the electric field in the \hat{z} direction due to a disk charge on the r plane as show in Fig L-1. The electric field in the \hat{z} direction is given by the following integral

$$\int_{r=0}^a \frac{\rho_s z}{2\epsilon_0} \frac{r dr}{(r^2 + z^2)^{3/2}},$$

Using function `integral()` written in the previous MATLAB exercise, compute the above integral by considering

$$f = \frac{z}{(r^2 + z^2)^{3/2}}$$
$$t = \frac{\rho_s r}{2\epsilon_0},$$

where $\rho_s = 2 \frac{\text{mC}}{\text{m}^2}$, $a = 10 \text{ cm}$, $z = [-2a, \dots, 2a]$, and ϵ_0 is the permittivity of free space. Then, compare the results with its analytical value

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2 + z^2}} \right) \hat{\mathbf{z}}.$$

Plot the symbolic and analytic solutions.

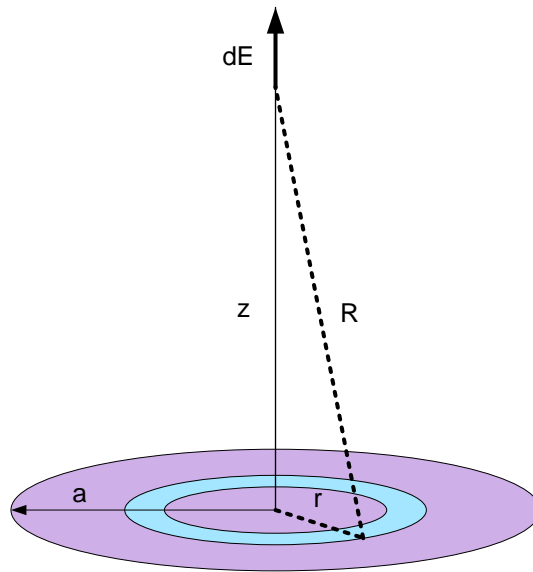


Fig L-1: Electric field due to a disk charge.

EXERCISE 3 (25 POINTS)

The purpose of this MATLAB exercise is to introduce numerical integration as the third (often the only available) way to solve integrals, besides symbolic MATLAB integration and analytical solutions. Consider the uniformly charged disk charge in Exercise 2. Evaluate this integral numerically for the electric field in the \hat{z} direction. The simplest numerical integration formula is

$$\int_a^b f(x)dx \approx \sum_{i=1}^N f(x_i)\Delta x, \quad \Delta x = \frac{b-a}{N} \quad (1)$$

where N denotes the number of integration segments (increments) and x_i , for $i = 1, \dots, N$ are the coordinates of **CENTER** segments. Plot the numerical solution.

EXERCISE 4 (25 POINTS)

Plot the square of the error between the numerical values and symbolic values for different values of N (the x-axis of the plot is N and the y-axis denotes the error).